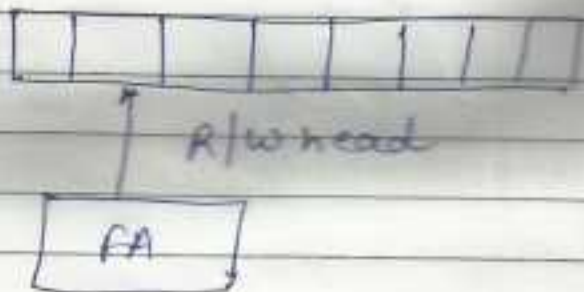


Turing Machine : It is a simple mathematical method of general purpose computers. It is capable of performing any calculation which can be performed by any calculating machine or a computing machine.



(Two-way FA) R/w head move in both direc.

In one transition machine examines present symbol under read-write head and present state of an automaton to determine the following:

- next state
- A new symbol to be written on the tape under R/w head
- movement of R/w head along the tape i.e. either left or right.

$Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$

↓

↓

$Q \times (\Sigma \cup \Gamma) \rightarrow$ input tape symbols

Turing Machine : It is a simple mathematical method of general purpose computers. It is capable of performing any calculation which can be performed by any calculating machine or and computing machine.



(Two-way FA) R/W head move in both dirce.

In one transition machine exam present symbol under read-write head and present state of an automaton to determine the fall

- next state
- A New symbol to be written on the table under R/W head
- movement of R/W head along the tape i.e either left or right.

$Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$
 $\downarrow \qquad \qquad \downarrow$
 $Q \times (\Sigma \cup \Gamma) \rightarrow$ input tape symbols

formal definition = $\langle Q, \Sigma, \Gamma, \delta, Q_0, b, F \rangle$
 where

$Q \rightarrow$ set of states

$\Sigma \rightarrow$ set of input

$\Gamma \rightarrow$ set of symbols to be written on tape

$Q_0 \rightarrow$ initial state

$b \rightarrow$ blank symbol $\in \Gamma$

$F \rightarrow$ set of final state

Representation of TM

1) By using ID'S

q, p, r

$\downarrow \quad \rightarrow$ present input
 current state

b a b' d f b

b	a	b'	d	f	b
---	---	----	---	---	---

\uparrow

q_3

$baq_3b'dfb$
 $q_1ba'b'dfb$

Left movement to right movement:

$(q, x_i) \vdash (p, y, L)$

$x_1 x_2 x_3 \dots x_{i-1} q x_i x_{i+1} \dots x_n \vdash$

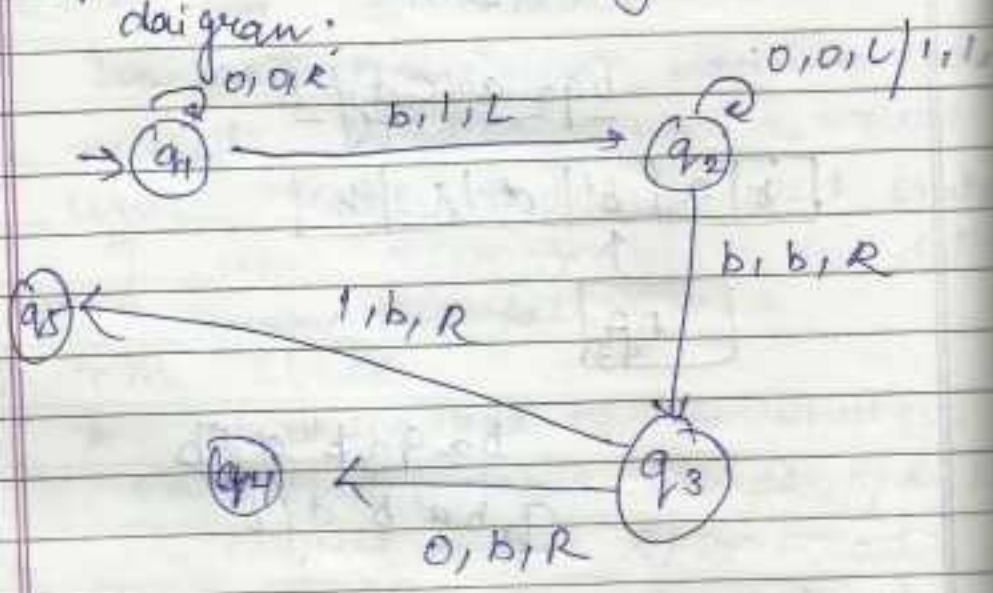
$x_1 x_2 x_3 \dots p x_{i-1} y x_{i+1} \dots$
 $(q, x_i) \vdash (p, y, R)$

Exd 2 Representation using transition table

	b	0	1
→ q ₁ Cq ₂	ORq ₁	-	
q ₂ BRq ₃	OLq ₂	1Lq ₂	
q ₃ -	ORq ₄	ORq ₅	
q ₄ ORq ₅	ORq ₄	1Rq ₄	
* q ₅ OLq ₂	-	-	

T B T
 ↓ ↓ ↓
 state
 Input movement

Representation using transition diagram



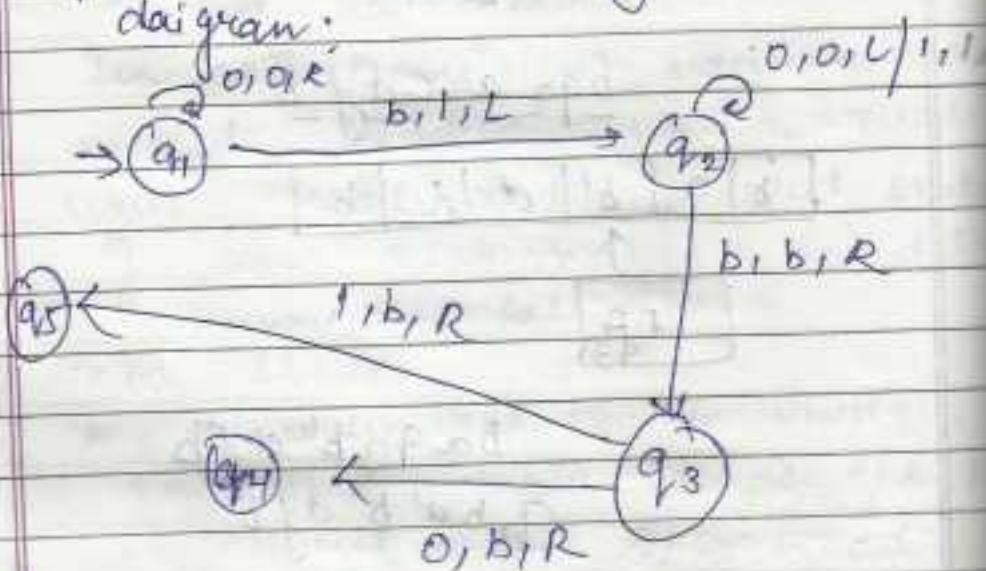
Ex 2

Representation using transition table

	b	0	1
$\rightarrow q_1$	$1q_2$	$0Rq_1$	-
q_2	bRq_3	$0Lq_2$	$1Lq_2$
q_3	-	bRq_4	bRq_5
q_4	$0Rq_5$	$0Rq_4$	$1Rq_4$
* q_5	$0Lq_2$	-	-

\uparrow B \uparrow Y
 \downarrow \downarrow \downarrow
 State
 Input Movement

Representation using transition diagram:



Language acceptability by T.M

Let M is equal to $\{Q, \Sigma, \Gamma, \delta, Q_0, b, F\}$
be a T.M.

A string w in Σ^* is said to
be accepted by M if $q_0 w b^* \xrightarrow{\delta^*} p_1 p_2$

$P \in F, \gamma_1, \gamma_2 \in (\Sigma \cup \Gamma)^*$

M doesnot accept w , if machine
in either halts in non-
accepting state or doesnot halt.

Halting problem of T.M - The
Halting prob of a T.M over the
input alpha Z is unsolvable
i.e the problem of deciding
whether or not a T.M
 M halts on arbitrary input
 w in Σ^* or not.

Unsolvable problem: A class of
problem with two output i.e
yes/no is said to be
solvable or decidable
if there exist some definite
algorithm which always
terminate with two output
i.e yes or no, then that
prob is decidable or solvable
otherwise problem is Undecide.

Language acceptability by T.M

Let M is equal to $\{Q, \Sigma, \Gamma, \delta, Q_0, b, F\}$
be a T.M.

A string w in Σ^* is said to
be accepted by M if $q_0 w p_1^* \vdash^* p_2$

$P \in F, \gamma_1, \gamma_2 \in (\Sigma \cup \Gamma)^*$

M doesnot accept w , if machine
in either halts in non-
accepting state or doesnot halt.

Halting problem of T.M - The
Halting prob of a T.M over the
input alpha Σ is unsolvable
i.e the problem of deciding
whether or not a T.M
 M halts on arbitrary input
 w in Σ^* or not.

Unsolvable problem: A class of
problem with two output i.e
yes/no is said to be
solvable or decidable
if there exist some definite
algorithm which always
terminate with two output
i.e yes or no, then that
prob is decidable or solvable
otherwise problem is Undecid.

Types of T.M

Deterministic

Non-Deterministic

 $IQ_1 \vdash IQ_2 \vdash IQ_3 \vdash \dots \vdash IQ_n$

⇒ Multiple head T.M
= K-dimensional

we can have 2D or 3D input

⇒ Universal TM - A general purpose T.M is called a Universal T.M. It is a machine that accepts two things + Input data, and description of computation i.e. algorithm. eg. computer.

check weather string $w = 0$ will be accepted by TM or not.

$q_1, 00b \vdash q_1, 0b \vdash 00q_1$
 $0, q_2 0b \vdash$

Process

We can add δ on starting and End.

$q_2 b q_0 | \text{---} | b q_3 @ 0 | \text{---} | b b q_4 0 |$
 $b b 0 q_4 | \text{---} | b b 0 1 q_4 b |$
 $b b 0 1 q_5 @ | \text{---} | b b 0 1 q_2 0 |$
 $b b 0 q_2 1 0 0 | \text{---} | b b q_2 0 1 0 0$
 $| \text{---} | b q_2 b 0 1 0 0 | \text{---} | b b q_2 0 1 0 0 |$
 $b b b q_4 1 0 0 | \text{---} | b b b 1 q_4 0 0 0 | \text{---} | b b b 1 0 q_4 0$
 $| \text{---} | b b b 1 0 0 q_4 b | \text{---} | b b b 1 0 0 0 q_5 b$
 $| \text{---} | b b b 1 0 0 q_2 0 0 | \text{---} | b b b 1 0 q_2 0 0 0 |$
 $b b b q_2 1 0 0 0 0 | \text{---} | b b q_2 b 1 0 0 0 0 |$
 $b b b q_3 1 0 0 0 | \text{---} | b b b b q_5 0 0 0 0$

modify Post Correspondence Problem

Ques:

PCP / MPCP

↓
Post Correspondence Problem

Consider two lists $X = x_1, x_2, x_3, \dots, x_n$

and $Y = y_1, y_2, y_3, \dots, y_n$, of non-empty strings over the input alphabet Σ , the PCP is

to determine whether or not i_1, i_2, \dots, i_n where $1 \leq i_j \leq n$ such that $x_{i_1} \dots x_{i_n} = y_{i_1} \dots y_{i_n}$

If there exist a solution to PCP then there exist infinitely many solutions

Example:

check whether that the PCP with two list has solution or not.

$$X = \{ b, bab^3, ba \}$$

$$Y = \{ b^3, ba, a \}$$

$$\begin{aligned} 2 \ b \ b \ a &= 2 \ 1 \ 1 \ 2 \\ &= bab^3 \ b \ b \ ba \\ &= \underline{bab^3 b b b a} \end{aligned}$$

$$\begin{aligned} &= (bab^3 a) \\ &= (bab^3 a)^2 \\ &= (bab^3 a)^3 \\ &= (bab^3 a)^4 \\ &= (bab^3 a)^5 \\ &= \end{aligned}$$

modify PCP of the first sub-
 [011111] use in PCP is X, and
 [011111] then that PCP is known
 as MPCP

~~10111~~
~~10111~~ Example: check whether
 PCP has solution or not.

$$\begin{array}{c|c} X & Y \\ \hline \textcircled{1} & 111 \\ \textcircled{10111} & 10 \\ 10 & 0 \end{array} = 2113$$

~~10111~~
~~10111~~

most have ans 2118

Date:

Page No.

$$= X \{01, 1, 11\}$$

$$= Y \{01^2, 10, 11\}$$

no
soln can not exist

$$X = \{01, 10, 11\}$$

$$Y = \{10, 01, 01\}$$

no soln exist.

$$X = \{10, 011, 101\}$$

$$Y = \{101, 11, 10\}$$

10110

10110

$$= 13$$

Church's Thesis:

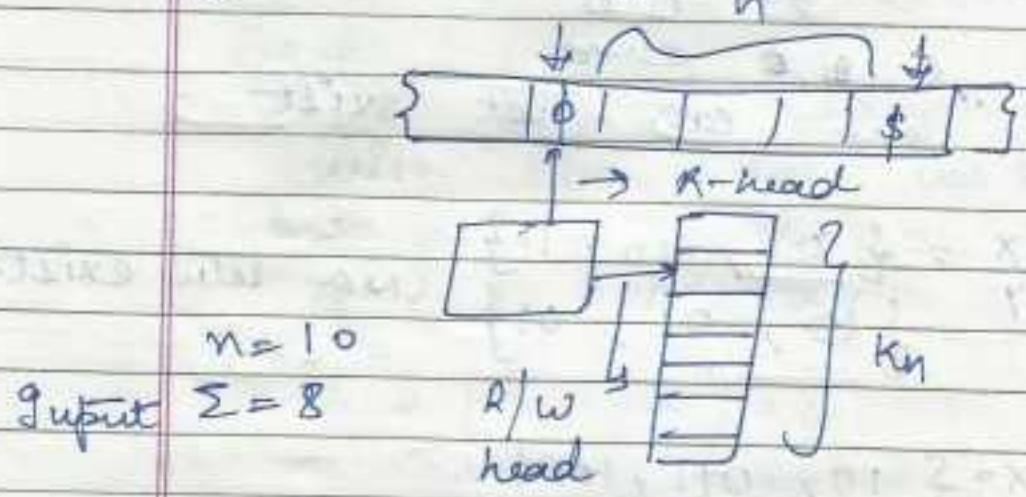
Statement: No computational procedure will be considered an algorithm unless it can be represented as TM.

This statement is known as Church's Thesis in 1936.

Linear Bounded Automata:

It is type-1 (context-sensitive lang.) LBA is non-deterministic TM which has a single Input Tape whose length is not ∞ and is bounded by a linear function.

LBA is formally define as
 $\{ Q, \Sigma, \Gamma, \delta, q_0, b, \phi, \$, F \}$



k is constant specified to programmer

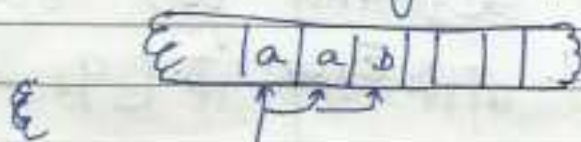
$$Q \times (\Sigma \cup \{ \phi, \$ \}) \times \Gamma \mid \rightarrow Q \times (LR) \times \Gamma$$

- $\Gamma \rightarrow$ storage tape
- $b \rightarrow$ blank
- $\phi \rightarrow$ start-marker
- $\$ \rightarrow$ End marker

$$Q \times (\Sigma \cup \{ \phi, \$ \}) \times \Gamma \mid \rightarrow Q \times (LR) \times \Gamma$$

Design of TM:

The state will be changed only when there is a change in the written symbol or there is change in the movement of r/w head.



- Q. Design a TM to recognize all the strings consisting equal all the string of 1 = $\Sigma^*(1)$

	1	b
$\rightarrow q_0$	q_0, bR	-
q_1	b, q_0	b, q_0, k

- Q. Design a TM which accept odd no of 1. - $\Sigma = \{1\}$

	1	
$\rightarrow q_0$	b, q_1, R	
q_1		b, q_0, R

- Q. Design TM for even no of 1, $\Sigma = \{0, 1\}$

	1	b	0
$\rightarrow *q_0$	bq_1R	-	$0q_0R$
q_1		bq_0R	$0q_1R$

Design a TM for odd no
 $q \perp = \Sigma(0,1)$ (q_1 will state)

Q. Design a TM compute the first complement

	1	0	b
$\rightarrow *q_0$	$0q_1R$	$1q_1R$	accept
$\rightarrow *q_1$	$0q_0R$	$1q_1R$	accept

Q. Design TM which compute
 concatenation function $\Sigma = 1$

	1	0	b
$\rightarrow q_0$	$1q_0R$	$1q_1R$	-
q_1	$1q_1R$	-	$1q_2R$
$*q_2$	$0q_2$	-	$0q_2$

q_2

$$Q = a^n b^n$$

$$a^n b^n c^n$$

equal no a and equal no b's

Q. Palindrome

q_0	a	b	x	y	Δ
$\rightarrow q_0$	xRq_1	yRq_2	-	-	
q_1	aRq_1	b	xRq_1 xLq_1	yRq_1	

q_0 ~~x~~ a a b b b

$a^n b^n$

q_0

q_0	a	b	x	y	Δ
q_0	xRq_1	-	-	yRq_0	ΔLq_3
q_1	aRq_1	yLq_2	-	yRq_2	
q_2	aLq_2	-	xRq_0	yLq_0	
* q_3	-	-	xLq_3	yLq_3	accept

Q. equal no of a's and b's

	a	b	x	y	Δ
$-q_0$	xRq_1	yRq_3	xRq_0	yRq_0	ϕLq_4
q_1	aRq_1	yLq_3	xRq_1	xRq_1	$\bar{y}Rq_1$
q_2	aLq_2	bLq_2	xLq_2	yLq_2	ϕRq_0

