BT-201 Roll No. 12176

B. TECH (CSE, IT, ECE, MAE, CE, E & EE, E & IE, AE, NST, S & AE), B. TECH (MAE) + M. TECH AUTOMOBILE ENGINEERING (DD), B. TECH (CSE, ECE, MAE, CE) + MBA DUAL DEGREE, B. TECH + M. TECH (NS & T) – DD & B. TECH (AE) + M. TECH (A) – DD

SECOND SEMESTER END TERM EXAMINATION

APRIL – 2013

APPLIED MATHEMATICS – II

Time : 3 Hrs *Maximum Marks : 70*

Note: ***Attempt questions from all sections as directed.***

SECTION – A (30 Marks)

Attempt any **5** *questions.*

Each question carries **6** *marks.*

1. Find values of λ, for which the following system of equations is consistent and has non-trivial solutions. Solve the equations for all such values of λ

$$(\lambda - 1)x + (3\lambda - 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

P.T.O.

2. Test whether the following vectors are linearly dependent ? If so, find a relationship between them.

$X_1 = (1, -2, 3, 4)^t$, $X_2 = (-2, 4, -1, 3)^t$, $X_3 = (-1, 2, 7, 6)^t$.

3. Test the convergence for the series

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \ldots$$

4. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ using Cauchy's Integral formula where C is $|z| = 3$.

5. Show that the function defined $f(z) = \sqrt{|xy|}$ satisfies Cauchy – Riemann equations at the origin but is not analytic at that point.

6. Write all possible Laurent series for the function

$$f(z) = \frac{1}{z(z+2)^3}$$

about the pole $z = -2$, using appropriate Laurent series.

SECTION – B (20 Marks)

Attempt any **two** *questions.*
Each question carries **10** *marks.*

7. (a) Show that the function $u(x, y) = 4xy - 3x + 2$ is harmonic. Construct the corresponding analytic function $x^2 + y^2 + z^2 = 1$. Express $f(z)$ in terms of z. (5)

(b) Show that the transformation $w = \frac{i - z}{i + z}$ maps the real axis of the z – plane into the circle $|w| = 1$ and upper half plane $y > 0$ into the interior of the unit circle $|w| < 1$ in the w - plane. (5)

8. Let $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$. Find the modal matrix P such that $P^{-1}AP$ is a diagonal matrix.

9. Discuss the convergence of the series

$$\frac{x^2}{2 \log 2} + \frac{x^3}{3 \log 3} + \frac{x^4}{4 \log 4} + \ldots$$

SECTION – C (20 Marks)

(Compulsory)

10. (a) Using Cauchy's residue formula, evaluate the following integral

$$\int_C \frac{4-3z}{z(z-1)(z-2)}\,dz \text{ where } C \text{ is a circle } |z| = \frac{3}{2}$$

(6)

(b) If the first four moments of a distribution about the value 5 are –4, 22, –117 and 560. Obtain (i) Mean (ii) Variance (iii) Skewness (iv) Kurtosis. (8)

(c) Show that Poisson Distribution is a limiting case of the binomial distribution. (6)